Axial-Symmetric Boundary Value Problem with Nonlinear Elasticity

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The application of solutions of linear elastic boundary value problems to the interpretation of the behavior of a number of real materials is a matter of expediency. For a number of materials, for instance, solid propellants, the assumption of a linear elastic stress strain relation is an approximation; even within the range of small strains, it is of interest to determine how the deviation from nonlinearity in a real material affects the stress distribution in a boundary value problem.

The formulation of any nonlinear response must be in invariant terms.[1] For a strain-softening material that has a linear volumetric response, a simple form of nonlinearity is obtained by assuming the shear modulus G to be a function of the second deviatoric strain invariant I_2', $G = G_0(1 - CI_2')$, where G_0 and C are constants that reflect the shear modulus as the magnitude of the strain approaches zero and the amount of the nonlinearity, respectively, and $I_2' = \frac{1}{2}e_{ij}\,e_{ij}$.

The authors[2] have investigated techniques for the solution of a nonlinear thick-walled cylinder (as just described) subjected to internal pressure and contained by a thin elastic shell with Young's modulus E under conditions of plane strain. Figure 1 shows some of the results of this investigation which do not appear in Ref. 2. These results were obtained by writing the equilibrium equation in terms of the displacement and subsequently applying finite difference techniques. An iterative procedure was programmed for a digital computer in which at each step the equation was linearized by determining the elastic constants from the previous step. For the largest pressure 60 iterations were used, which required about two hours on an IBM 1620 when the cylinder was divided into 40 sections.

Figure 2 shows the relationship between the second deviation stress invariant, $J_2' = \frac{1}{2}\,s_{ij}\,s_{ij}$, and I_2'. This curve is very nearly linear for small values of I_2' for which the behavior of the cylinder differs only slightly from the behavior of a linear cylinder. It is seen (Fig. 1) that, for increasing pressure, the nondimensional circumferential stress near the inside surface decreases in a manner quite similar to the elastic-plastic cylinder. However, for larger pressures the stress becomes negative and there is a significant corresponding increase in longitudinal stress (not shown). For increasing pressure, the material approaches a state of hydrostatic stress.

In the example shown $a/b = 0.5$, $t/b = \frac{1}{40}$, $E/G_0 = 100$, $K_0/G_0 = 1$, and $C = 2 \times 10^6$.

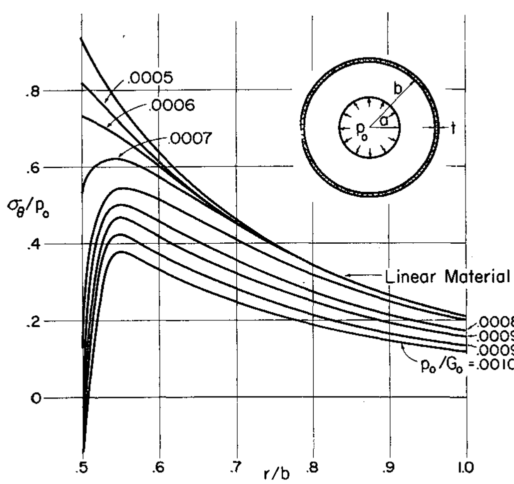

Fig. 1 Circumferential stress diagrams

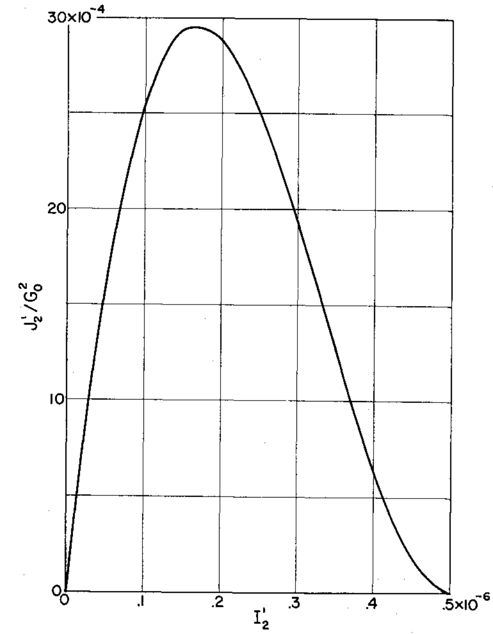

Fig. 2 Deviatoric stress invariant vs deviatoric strain invariant

References

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[2] Spillers, W. R., "Two solutions for the non-linear elastic thick walled cylinder under pressure," Office of Naval Research TR11, CU-11-61, ONR 266(78) CE (February 1962).

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Electromagnetic Probe for the Measurement of Hypersonic Flow Velocity at a Point

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In the past it has not been possible to measure freestream velocity at a point in a hypersonic flow field without undue difficulty. At the Naval Ordnance Laboratory, a feasibility study was made to determine whether such a velocity-measuring technique could be developed. A method was devised to make use of a rapidly varying high-intensity magnetic field in the stagnation region about a probe to produce a disturbance in the standing shock. This disturbance is observable by high-speed schlieren photography as it travels outward along the standing shock.

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